FM A2 U5 FMSP 2 MS

1

(a)	$E(X) = -\theta + 1 - 3\theta = 1 - 4\theta$	B1
	$\text{Var}(X) = \theta + 1 - 3\theta - (1-4\theta)^2$	M1
	$= \theta + 1 - 3\theta - 1 + 8\theta - 16\theta^2$	A1
	$= 2\theta(3 - 8\theta)$	
(b)	$E(U) = \dfrac{1 - E(\overline{X})}{4}$ [M1A0 if E omitted]	M1
	$= \dfrac{1-(1-4\theta)}{4}$	A1
	$= \theta$	
	$\text{Var}(U) = \dfrac{\text{Var}(\overline{X})}{16}$	M1
	$= \dfrac{2\theta(3-8\theta)}{16n}$	A1
(c)	N is $B(n, 2\theta)$; $E(N) = 2n\theta$ si	B1
	$E(V) = \dfrac{2n\theta}{2n} = \theta$ [B0 if E omitted]	B1
	$\text{Var}(N) = 2n\theta(1 - 2\theta)$ si	B1
	$\text{Var}(V) = \dfrac{\text{Var}(N)}{4n^2}$	M1
	$= \dfrac{\theta(1-2\theta)}{2n}$	A1
(d)	$\text{Var}(V) - \text{Var}(U) = \dfrac{1}{n}\left(\dfrac{\theta}{2} - \theta^2 - \dfrac{3\theta}{8} + \theta^2\right)$	M1
	$= \dfrac{\theta}{8n}$ (> 0)	A1
	[FT from previous results]	
	U is better because $\text{Var}(U) < \text{Var}V)$	B1

2

3(a)	Upper quartile = 40 + 0.674(5) × 2.5	**M1**	M0 no working
	= 41.7	**A1**	
(b)(i)	Let X=weight of a male, Y=weight of a female		
	Let $U = X_1 + X_2 + X_3 + Y_1 + Y_2$		
	$E(U) = 3 \times 40 + 2 \times 32 = 184$	**B1**	
	$\text{Var}(U) = 3 \times 2.5^2 + 2 \times 1.5^2 = 23.25$	**B1**	
	$z = \dfrac{185 - 184}{\sqrt{23.25}} = 0.21$	**M1A1**	
	Prob = 0.4168	**A1**	Accept 0.417
(ii)			
	Let $W = X_1 + X_2 + X_3 - 2(Y_1 + Y_2)$	**M1**	
	$E(W) = 3 \times 40 - 4 \times 32 = -8$	**A1**	
	$\text{Var}(W) = 3 \times 2.5^2 + 8 \times 1.5^2 = 36.75$	**M1A1**	
	$z = \dfrac{8}{\sqrt{36.75}} = 1.32$	**m1A1**	
	Prob = 0.9066	**A1**	Accept 0.907

3

(b)(i)	$\mu < 10000$	B1	1	
(ii)	$n = 16$ and $s = 500$; $t_{crit} = 1.753$	B1		For t_{crit} (ignore signs)
	$\text{sd}(\bar{X}) = \frac{500}{\sqrt{16}}$ (125)	B1		Ignore notation
	Critical value is one of: $10000 \pm 1.753 \times \frac{500}{\sqrt{16}}$ (considered)	M1		M0 if only considered upper value No ft on incorrect t value
	Choose 9780 (3sf)	A1		AWFW 9780 to 9781 (ignore inequality)
	($\Rightarrow$ critical region: $\bar{x} < 9780$)			If z used then max B0B1M0A0A0
	$\therefore$ Range of values for $\bar{x}$ which leads Christine **not** to reject H_0: $\mu = 10000$ is: $\bar{x} > 9780$	A1	5	Allow $\bar{x} \geq 9780$ to 9781
(iii)	No error	B1	1	Ignore any subsequent statements

4

(a) $H_0 : \mu_G = \mu_B$ versus $H_1 : \mu_G \neq \mu_B$ B1

(b) $\Sigma g = 105.1$, $\Sigma b = 86.7$ or $\bar{g} = 13.1375, \bar{b} = 14.45$ B1

The appropriate test statistic is

$$\text{TS} = \frac{\bar{g} - \bar{b}}{\sigma\sqrt{\frac{1}{m} + \frac{1}{n}}} \qquad \text{M1}$$

$$= \frac{105.1/8 - 86.7/6}{1.5\sqrt{\frac{1}{8} + \frac{1}{6}}} \qquad \text{A1A1}$$

$$= -1.62 \qquad \text{A1}$$

tabular value = 0.0526 A1

p-value = 0.1052 B1

(c) Insufficient evidence to conclude that the means are different. B1

5

(a)	H_o pop median/mean diff $\eta_d = 0$	B1		
	H_1 pop median/mean diff $\eta_d < 0$	B1		Consistent with differences
	1 tail 5% (d is 2003 – 1999)			
	diff: -5.4, -3.2, -3.8, -4.2, -2.4; **rank**: 10, 6, 8, 9, 3	M1		For differences
	-2.1, -3.1, +0.3, -2.8, +3.4; ranks: 2, 5, 1, 4, 7	M1		For ranks
	$T_+ = 1 + 7 = 8$	m1		For total
	$T_- = 10 + 6 + \ldots + 4 = 47$	A1		For one correct total
	ts T = 8 $n = 10$ cv = 11	B1		For cv
	T < 11	M1		Comparison cv/ts
	Significant evidence at 5% level to reject H_o and conclude that average teenage conception rate has decreased between 1999 and 2003	E1	9	In context
(b)	A matched pairs design eliminates individual differences by comparing conception rates in the same regions for the two years.	B1		General idea of matched pairs reducing experimental error
	This means that any particular regional differences will not affect the comparisons and so a difference is more likely to be detected if one exists	E1	2	In context
(c)	A Type I error is when a correct H_0 is rejected.	B1		
	In this case it would mean that we conclude that the average conception rate has decreased when, in fact, it has not	E1	2	
	Total		**13**	

6

H_0 Samples are taken from identical populations H_1 Samples are not taken from identical populations 2 tails 5%	B1		H_0 $\eta_M = \eta_A$ or words ref H_1 $\eta_M \neq \eta_A$ context Disallow mean
Separated times with Ranks	M1 M1		Separated times effort (can be implied) Ranks as one group (either way)
(table below)	A1		Ranks correct (5,6 or 9,10 OK)
$T_M = 66$ 39 $T_A = 39$ 66 $n_M = 7$ $n_A = 7$	m1		Ranks totalled (any ranks) m dep ranks
$U_M = 66 - \frac{7\times 8}{2} = 38$ $U_A = 39 - \frac{7\times 8}{2} = 11$	m1 A1		Attempt to find U dep ranks, totals Either U correct
$U = 11$ cv = 9 for $n = 7$, $m = 7$ 2 tail 5%	B1		cv correct cv = 9 only
$U > 9$	M1		correct comparison, ft on wrong ts – must see 11/lower U oe upper tail unless all correct
Accept H_0	A1		only if cv = 9 and $U = 11$
No significant evidence of any difference between average journey times when travelling for the morning shift or for the afternoon shifts	E1	11	In context. Can ft conclusion
Total		**11**	

M		A	
Times	ranks	Times	ranks
19.2	1 14	21.3	4 11
22.4	8 7	22.3	7 8
26.8	13 2	19.6	2 13
22.5	9 6	20.2	3 12
24.8	11 4	21.7	5½ 9½
24.6	10 5	21.7	5½ 9½
28.4	14 1	26.2	12 3

7

3(a)	$\hat{p} = \frac{654}{1500} = 0.436$ si	**B1**	
	ESE = $\sqrt{\frac{0.436\times 0.564}{1500}} = 0.0128..$ si	**M1A1**	
	95% confidence limits are $0.436 \pm 1.96 \times 0.0128..$ giving [0.41,0.46]	**M1** **A1** **A1**	M1 correct form A1 correct z
b)	$\hat{p} = \frac{0.4348+0.4852}{2} = 0.46$	**B1**	
	Number of people = 0.46 × 1200 = 552	**B1**	
	$0.4852 - 0.4348 = 2z\sqrt{\frac{0.46\times 0.54}{1200}}$	**M1A1**	
	$z = 1.75$ Prob from tables = 0.0401 or 0.9599 Confidence level = 92%	**A1** **A1** **B1**	FT line above